A Unified Interference/Collision Analysis for Power-Aware Adhoc Networks

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Abstract— In this paper we address the issue of controlling transmission power in power-aware adhoc networks. Previous work that minimizes the transmission power does not consider both the energy consumed in collision resolution and the energy disbursed to overcome the interference resulting from neighboring nodes. We investigate the basic transmission power control for the 802.11 MAC protocol, in which the control frames and the data frames can be transmitted at different power levels. A collision model together with an interference model of a uniformly distributed network are constructed. Based on these models, the end-to-end network throughput and the total energy consumption of the network are examined. For a network with a given node density, our results show the optimal transmission power for control messages and for data messages that will yield maximum throughput and minimum energy consumption per message.

I. INTRODUCTION

Adhoc networks have witnessed an explosion of interest in the last few years as they are expected to have a significant impact on the efficiency of many military and civilian applications, such as combat field surveillance, security and disaster management, data gathering, and conferences. An adhoc network is an infrastructureless multihop wireless network in which all devices establish direct communication with other nodes without a centralized entity. One of the constraints for building an efficient adhoc network is finite battery supplies. Since the network nodes are battery operated, and in many cases they are installed in an environment where it may be impossible to retrieve the nodes in order to recharge the batteries, the network nodes need to be energy conserving so that the battery life and hence the network lifetime (total time in which the network is connected and functioning) are maximized. Recent research addressed this challenge and various approaches are proposed for each layer of the communication protocol stack [15] to reduce the energy consumption.

Some previous work [17] [8] proposed the idea of minimizing the transmission power and sending the data in a multi-hop fashion to the destination by relaying the packets at intermediate closer nodes. Although the transmission energy is reduced by such scheme, the effect of transmission power control schemes on the total network throughput and the overall energy consumption were not investigated.

Our work is based on the observation that there is a tradeoff in the choice of the transmission power. When reducing the transmission power, the number of nodes included within the transmission range of the sender and competing for wireless channel access is reduced and hence the number of collisions is reduced. However, at every relay node, the data message is relayed and forwarded, consequently, the probability of collision per message is increased. As a result, in the multihop scheme, collision resolution may end up using more energy than the one hop direct transmission scenario. On the other hand, with respect to interference, it is intuitive that using reduced power minimizes the interference level between neighboring nodes. However, there is an increase in the number of concurrent transmissions because the transmission range of each node is reduced. Consequently, the overall Signal to Interference Ratio (SIR) might degrade when using a lower transmission power.

In this paper, by taking into consideration the energy wasted in the collision resolutions and the energy used to overcome the interference signal level of neighboring nodes, we argue that the minimum transmission power will not always deliver an optimal energy consumption. We investigate the transmission power adjustment problem to minimize the energy consumption of an adhoc network, based on the 802.11 (CSMA/CA) MAC protocol. An analytical collision model together with an interference model are both constructed for a uniformly distributed adhoc network. From these models we were able to derive the total network throughput and the total energy consumption in the network.

The rest of the paper is organized as follows: Section II presents related work and different power control schemes. Section III discusses the background and main assumptions. Sections IV and V describe the interference
and the collision models. Section VI analyzes the total energy consumption in the network. Numerical results are presented in Section VII. We conclude the paper in Section VIII.

II. RELATED WORK

Recognizing the challenge of energy consumption in ad-hoc networks, much research has been directed toward the design of energy aware protocols. We can categorize the previous research work on power-aware MAC layer into three categories, Reservation Based Power-Aware MAC, Switching Off Power-Aware MAC and Transmission Power Control.

The Reservation Based Power-Aware MAC tries to avoid collisions in the MAC layer, since collisions may result in retransmissions, leading to unnecessary power consumption. The EC-MAC [26], presented the idea of applying reservation schemes in wireless networks MAC protocols for energy conservation. Although EC-MAC was originally constructed for networks with base stations serving as access points, its definition could be extended to adhoc networks, where a group of nodes may select some type of coordinator to perform the functions of a base station, as proposed in [2] and [22]. Furthermore, because the coordinator can consume the resources of certain nodes, a group of schemes were proposed in which the coordinators are rotated among network nodes. In [11] the coordinators are randomly chosen while in [10] the remaining battery capacity controls the probability of coordinator selection.

The Switching off Power-Aware MAC tries to minimize the idle energy consumption by forcing nodes to enter the doze mode. For example, PAMAS [25], allows a station to power its radio off when it has no packet to transmit/receive but has to keep a separate channel on which the RTS/CTS packets are received. Similarly, Chiasserini [3] allows a station to go to sleep, but a special hardware, called Remote Activated Switch (RAS), is required to receive wake-up signals. Also, in [31] the geographical area is partitioned into smaller grids in each of which only one host needs to remain active to relay packets for all the stations in the same grid. Furthermore, Pattern [21], discussed various activation strategies for the nodes, including a randomized way and another one based on the activity region.

Since the maximum power in the wireless card is consumed during the transmission mode, much research has been proposed to minimize the transmission power and thus maximize the network lifetime. For example, PARO [8] sends the data to the nearest neighbor in a multihop fashion until reaching the destination. Furthermore, the control frames (RTS/CTS) are sent with maximum power, while the data and acknowledgment frames are sent with reduced power, as will be discussed in the next section. Other protocols control the transmission power not only based on the distance between the sender and the receiver but also based on different channel conditions. For example, the scheme presented in [23] adjusts the transmission power according to the SNR at the receiver. It allows a node, A, to specify its current transmit power level in the transmitted RTS, and allows the receiver node, B, to include a desired transmit power level in the CTS sent back to A. Analogously, the protocol in [5] chooses an appropriate transmission power based on the packet size.

III. MODEL BACKGROUND

Many previous works have made different assumptions about the radio characteristics of the wireless interface cards, including energy dissipation in transmit, receive, idle and doze modes. Detailed measurement results reported in [4] and [6] emphasized that the maximum power is consumed in the transmit mode. However, if the transmission/receive durations are small relative to idle time (a typical sensor networks environment), controlling only the transmission power may not be the most appropriate way to save energy rather than putting nodes to sleep.

In our work we only analyze the transmission power control schemes because (1) an adhoc network application is different in nature from a sensor network, (2) a considerable portion of the adhoc network lifetime is typically consumed in transmitting and receiving data between nodes, and (3) the maximum power is consumed in the transmit mode.

According to the path-loss radio propagation model, the ratio between the received signal power, $P_{Rx}$, at distance $r$ from the transmitter, to the transmitted signal power, $P_{Tx}$, is given by:

$$\frac{P_{Rx}}{P_{Tx}} = C \cdot r^{-\gamma}$$

(1)

where $C$ is a constant that depends on the antenna gains, the wavelength, and the antenna heights, $r$ is the transmission distance, and $\gamma$ is the path loss factor, ranging from 2 (line of sight free space) to 4 (indoor) [16].

In our network model, we assume that a set of homogeneous adhoc nodes are uniformly distributed over a large two dimensional area and the node density is given by $\rho$ per unit area. Each node can communicate and receive data directly from all the nodes within its coverage area, where the coverage area of the node is defined by the radius which the control frames can reach (defined as $a_{RTS}$). The MAC layer used in such communication is the CSMA/CA protocol with sender-initiated 4-way handshaking scheme, as defined in the 802.11 IEEE standard DCF MAC operation [14]. The
transmission of a data packet and its acknowledgment is preceded by request-to-send (RTS) and clear-to-send (CTS) packets between a pair of sending and receiving nodes, other nodes that overhear RTS or CTS packets will defer their access to the channel to avoid collisions. Based on the uniformly distributed nodes model, all the network hosts will use the same transmission power for DATA/ACK frames and thus will reach the same transmission range defined as \( a_{data} \). Similarly, all hosts use the same power for transmitting the control frames and this has the same coverage area defined by \( a_{RTS} \) (which can be different from \( a_{data} \)).

Furthermore, we will assume that the time is slotted with slot time \( \tau \). We define the number of time slots needed to send an RTS packet as \( L_{RTS} \) slots. Analogously, the number of time slots needed to send a CTS, a data packet, and an acknowledgment packets are \( L_{CTS} \), \( L_{data} \), and \( L_{ack} \), respectively.

As can be observed from Equation (1), to minimize the transmission energy consumption, it is always better to send the data in a multi-hop fashion using relay nodes rather than sending it directly to the destination. A simple power control scheme for the 802.11 RTS/CTS protocol should adjust the transmission energy for data and control frames (RTS/CTS) according to the distance between the sender and the relay node. However, as shown in Figure 1, different power levels among network nodes introduce asymmetric links, a problem known as the “Hidden Terminal Jamming” problem [30]. A hidden node C not sensing an ongoing low power data transmission, can corrupt the data packets being sent from A to B by concurrently transmitting a message to node D. Therefore, as depicted in Figure 2, the control frames have to be transmitted using a high power level, while the DATA and ACK are transmitted using the minimum power level necessary for the nodes to communicate [7] [23].

The expected number of hops, \( \bar{H} \), needed between any source and any destination node is given by:

\[
\bar{H} = \lfloor \frac{\bar{L}}{a_{data}} \rfloor \tag{2}
\]

where \( \bar{L} \) is the average path length of a message in the adhoc network and \( a_{data} \) is the radius by which the DATA/ACK packets are sent, that is, the distance between two consecutive relay nodes. Estimating the expected path length, \( \bar{L} \), is a function in the node distribution, dynamic patterns of mobility and traffic patterns in the network [19] [20].

Li et al. [18] provide a mathematical formula to calculate the expected path length between any source destination pair in an adhoc network. This length is given as a function of the total network coverage area and a locality index of the traffic. They analyzed two traffic patterns, a uniform random traffic in which a source chooses its destination with equal probability, and a local traffic pattern in which it is most probable that a node communicates with a near host rather than a further one.

### IV. Interference Model

Gupta and Kumar [9] showed that the transmission capacity of an adhoc network is inversely proportional to the square root of the number of nodes in the network due to the increased number of collisions. A collision, as defined by IEEE 802.11, occurs when two or more nodes within the sender coverage area transmits RTS packets at the same time or when an RTS collides with the CTS sent by the receiver node. Collisions can only occur during what is called Contention Window [14].

Further, the network throughput is also affected by the interference level caused by hosts concurrently sending their data. Interference occurs during the transmission time of a data frame, where nodes outside the RTS sensing area of the sender and the CTS sensing area of the receiver may concurrently transmit causing a background interference signals that degrades the Signal to Interference Ratio (SIR), causing an increase in the Bit Error Rate (BER).

The degradation in the total network throughput caused by a low SIR can be a serious problem. We extend the honey grid model defined in [12], with a new interference model for an adhoc network. We use this model to determine an upper bound on the total injected traffic by each node in the network.

Since nodes defer sending any packets upon hearing an RTS/CTS control frame, there will be no source of interference within the node’s coverage area. As shown...
in Figure 3, when Node 0 is transmitting, there will be no interference from any other node within $a_{RTS}$ from it. In the worst case, the first interfering node is just outside the coverage area of Node 0 (e.g., Node 1 at distance $a_{RTS} + \varepsilon$ from Node 0). The next interferer could only be outside the coverage areas of both nodes, and in the worst case at the crossing point of two circles each with radius $a_{RTS} + \varepsilon$. The constellation of interfering nodes is as shown in Figure 3.

Furthermore, for the worst case scenario of signals interfering with the data packet currently being received at Node 0 there are at most 6 interfering nodes at distance $a_{RTS} + \varepsilon$, and on the next interfering ring, at distance $2 \cdot (a_{RTS} + \varepsilon)$, there are at most 12 interfering nodes and so on. This results in the Honey Grid Model, depicted in Figure 4.

Assume that the “own” traffic generated by each node is $\mu$ messages per second, and on average there are $(H - 1)$ relay nodes between any source and destination pair. Therefore, the expected volume of relay traffic reaching any node is given by $\mu \cdot (H - 1)$. Consequently, the total traffic per node can be given:

$$\text{total traffic per node} = \text{own traffic} + \text{relay traffic} = \mu + \mu \cdot (H - 1) = \mu \cdot H$$

In order to get an upper bound on the own traffic produced by each node and injected into the network, $\mu$, we compute the worst case interference scenario, that is when all the interferers are actively transmitting. We add the received interference power from 3 nodes in the first ring at distance $a_{RTS}$, and 6 nodes in the second ring at $2a_{RTS}$, and so on. Since the network is uniformly distributed, we can assume that all the data/ack packets are sent with signal level $P_{data}$ covering a radius of $a_{data}$. On the other hand, the control frames are sent with a high power covering a radius of $a_{RTS}$. From Equation (1), for a fixed Bit Error Rate, the ratio between the control packets transmission power to the data packets transmission power is equal to the ratio of distances raised to the power of $\gamma$. Hence, the power by
which the control frames are sent, $P_{\text{RTS/CTS}}$ is given as:

$$P_{\text{RTS/CTS}} = P_{\text{data}} \cdot \left( \frac{a_{\text{RTS}}}{a_{\text{data}}} \right)^\gamma$$  \hspace{1cm} (4)

where $\gamma$ is the path loss factor (see Equation (1)).

Let $T_{\text{total}} = L_{\text{RTS}} + L_{\text{CTS}} + L_{\text{data}} + L_{\text{ack}}$ be the total time to send one frame (without any retransmissions). Then the interference level, $I_r$, of a single interferer located at distance $r$ from the receiving node is

$$I_r = q \cdot \left( P_{\text{data}} \cdot r^{-\gamma} \cdot \frac{L_{\text{data}} + L_{\text{ack}}}{T_{\text{total}}} \right) + P_{\text{data}} \cdot \left( \frac{a_{\text{RTS}}}{a_{\text{data}}} \right)^\gamma \cdot r^{-\gamma} \cdot \frac{L_{\text{RTS}} + L_{\text{CTS}}}{T_{\text{total}}}$$  \hspace{1cm} (5)

where $q$ is the probability of transmission per node. The first term inside the brackets represents the interference level caused by the data/ack packets with power $P_{\text{data}}$, and the second term accounts for sending the control frames (RTS/CTS) with the power defined in Equation (4).

Using Equation (5), we can compute the total interference at Node 0 caused by other network nodes in the honey grid model as:

$$I = \frac{3 \cdot q \cdot P_{\text{data}} \cdot a_{\text{RTS}}}{T_{\text{total}}} \sum_{i=1}^\infty \left\{ \left( e^{-(\gamma-1)} \times \left( 1 + \frac{a_{\text{RTS}}}{a_{\text{data}}} \right)^\gamma \right) \cdot \left( L_{\text{RTS}} + L_{\text{CTS}} \right) \right\}$$  \hspace{1cm} (6)

This is done by substituting distance $r$ with the radius of the $i^{th}$ interfering ring and summing up for all $3i$ interfering nodes in this ring. Since the series in Equation (6) is a converging series, the interference level caused by a distant node can be neglected if it is below a certain threshold which depends on the type of the interface card used.

The SIR at the Node 0 can be derived as the ratio between the signal level of the sender at distance $a_{\text{data}}$ away from Node 0 to the total interference level at this node, as defined by Equation (6). Hence, the SIR can be given as:

$$SIR = G \cdot \frac{P_{\text{data}} \cdot a_{\text{data}}^{-\gamma}}{I}$$  \hspace{1cm} (7)

where $G$ is the spread spectrum “Processing Gain” [24] used in the network physical layer.

Assuming that the total traffic per node is a Poisson process and that $\bar{H}$ is given in Equation (2), then the probability that a node transmits, $q$, is given as:

$$q = 1 - e^{-\mu \cdot \bar{H}}$$  \hspace{1cm} (8)

By substituting $q$ in Equation (6) and then substituting back the total interference level, $I$, in Equation (7), then rearranging the equation, the maximum traffic that a node can produce, $\mu$, while keeping $SIR = SIR_{\text{min}}$ at all other nodes, is:

$$\mu = \frac{-a_{\text{data}}}{L} \cdot \ln \left[ 1 - \frac{T_{\text{total}} \cdot G \cdot a_{\text{data}}^{-\gamma} \cdot \sum_{i=1}^\infty (\frac{a_{\text{RTS}}}{a_{\text{data}}})^\gamma \cdot (L_{\text{RTS}} + L_{\text{CTS}})}{3 \cdot SIR_{\text{min}} \cdot a_{\text{RTS}} \cdot \sum_{i=1}^\infty (\frac{a_{\text{RTS}}}{a_{\text{data}}})^\gamma \cdot (L_{\text{RTS}} + L_{\text{CTS}})} \right]$$  \hspace{1cm} (9)

As illustrated in Section VII, $\mu$ will be used to derive and evaluate the total network throughput. The network throughput is defined as the sum of the throughputs of each node that can concurrently transmit without causing a collision. Evaluating the total throughput at different values for both $a_{\text{data}}$ and $a_{\text{RTS}}$ will demonstrate the presence of a certain optimum transmission range for the control and data messages at which the throughput is maximized.

V. COLLISION MODEL

The nodes included within the coverage area of a certain host may send control messages that collide with the RTS/CTS frames transmitted by this node. A collision resolution scheme (exponential backoff) [13] is applied whenever a collision is detected. The higher the number of collisions, the lower the network throughput and the higher the energy consumed resolving these collisions. We modify and apply the collision model proposed in [29] for a uniformly distributed multihop adhoc network, and using this model, we derive the effect of collisions on both the throughput and the total energy consumption.

![Wireless Channel State Transition Diagram](image)

The wireless channel state transition diagram around a certain node $x$ is shown in Figure 6. IDLE is the state when channel around node $x$ is sensed idle, and its duration is for one time slot, $\tau$. The Transmit state indicates that a successful four-way handshake is completed, and hence, its duration is $T_{\text{transmit}} = L_{\text{RTS}} + L_{\text{CTS}} + L_{\text{data}} + L_{\text{ack}}$.
The \textit{RTS-col} state indicates that multiple hosts within the coverage area of node \( x \) transmit RTS frames concurrently, causing an RTS collision; its duration is \( T_c = L_{RTS} \). Finally, the \textit{CTS-col} state indicates that a terminal hidden from node \( x \) sends some packets that collide at the receiver with the RTS being received or the CTS being sent; its duration is \( T_c = L_{RTS} + L_{CTS} \).

In our analysis, we assume that the size of the \textit{Contention Window} (CW) is held constant. As proved in [13] and [1], the probability that a fully saturated node, a node that is always having a packet waiting in the output buffer to be sent, transmits at a given time slot, \( p \), is given by

\[
p = \frac{2}{CW + 1}
\]  

(10)

Using \( p \) we can derive the transition probabilities for the collision model as follows. The probability \( P_{II} \) is the transition probability from \textit{IDLE} to \textit{IDLE}, that is, the probability that none of the nodes within the coverage area of \( x \) transmits at this time slot. \( P_{II} \) is given by:

\[
P_{II} = (1 - p)^M
\]  

(11)

where \( M = \rho \cdot \pi a_{RTS}^2 \) is the total number of nodes included in the coverage area of node \( x \).

The probability \( P_{it} \) is the transition probability from \textit{IDLE} to \textit{Transmit}. It is the probability that exactly one node transmits at this time slot and starts a successful four-way handshake (i.e., other nodes withhold their transmission). \( P_{it} \) is given by:

\[
P_{it} = \frac{M}{1} \cdot \Pi_i \cdot (1 - p)^{M-1}
\]  

(12)

where \( \Pi_i \) denotes the probability that a node begins a successful four-way handshake at this time slot. \( \Pi_i \) is a function of the number of hidden terminals and the distance between the sender and the receiver as will be discussed later in this section.

The probability \( P_{ir} \) is the transition probability from \textit{IDLE} to \textit{RTS-col}. It is the probability that more than one node transmits an RTS packet at the same time slot. In other words, \( P_{ir} \) is \( (1 - \text{probability that none of the nodes transmits} - \text{probability that exactly one node transmits})\):

\[
P_{ir} = 1 - (1 - p)^M - M \cdot p \cdot (1 - p)^{M-1}
\]  

(13)

Finally, \( P_{ic} \), the transition probability from \textit{IDLE} to \textit{CTS-col}, can be simply computed as:

\[
P_{ic} = 1 - P_{it} - P_{ih} - P_{ir}
\]  

(14)

Having calculated \( P_{II}, P_{it}, P_{ir} \) and \( P_{ic} \), the equilibrium equations of the wireless channel state transition diagram can be deduced and solved, so that the \textit{Transmit} state limiting probability, \( \theta_t \), can be computed. \( \theta_t \) represents the percentage of time in which the node is successfully transmitting, or in other words, it is the ratio between successful transmission time to the total network time (defined as the summation of transmission time and contention time). The solution of the state model equilibrium equations is:

\[
\theta_t = \frac{P_{it}}{1 + P_{it} \cdot T_{transmit} + P_{ir} \cdot T_r + P_{ic} \cdot T_c}
\]  

(15)

All the terms of Equation (15) have been derived with the exception of \( P_{it} \); as it depends on \( \Pi_i \), the probability that a node starts a successful four-way handshake in the given time slot. In order to determine, \( \Pi_i \), the state transition diagram of a wireless node is constructed as shown in Figure 7. Node \( x \) is in the \textit{succeed} state when it can complete a successful four-handshake with the other nodes, and it enters the \textit{fail} state when the node initiates an unsuccessful handshake. On the other hand, the \textit{wait} state accounts for deferring for other nodes. \( \Pi_i \) is the limiting probability of the \textit{succeed} state, as computed next.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig7}
\caption{Wireless Node State Transition Diagram}
\end{figure}

We define \( B(a_{data}) \) to be the hidden area from node \( x \) when communicating with node \( K \) located at \( a_{data} \) away from it, as illustrated in Figure 8. Takagi [27] has proved that \( B(a_{data}) \) takes the form:

\[
B(a_{data}) = \pi \cdot a_{RTS}^2 - 2 \cdot a_{RTS}^2 \cdot \arccos \left( \frac{a_{data}}{2 \cdot a_{RTS}} \right)
\]  

\[
- \frac{a_{data}^2}{2 \cdot a_{RTS}} \cdot \sqrt{1 - \frac{a_{data}^2}{4 \cdot a_{RTS}^2}}
\]  

(16)

The number of nodes hidden from the sender, computed as \( \rho \cdot B(a_{data}) \), are not included in the sender coverage area but are within the receiver node coverage and can collide with the RTS frame being received or the CTS frame transmitted by the receiver.

The transition probability \( P_{ww} \), from \textit{wait} state to \textit{wait} state, is the probability that neither node \( x \) nor any node within its coverage area is initiating any transmissions. \( P_{ww} \) is given by:

\[
P_{ww} = (1 - p)^M
\]  

(17)
The transition probability, $P_{ws}$, from wait state to succeed state is the probability that node $x$ transmits at this time slot and none of the terminals within $a_{rts}$ of it transmits in the same slot, and also that none of the hidden nodes in $B(a_{data})$ transmits for $(2L_{rts})$ slots. $P_{ws}$ can be written as:

$$P_{ws} = p \cdot (1-p)^M \cdot ((1-p)^p B(a_{data}))^{2L_{rts}} \quad (18)$$

Finally, the transition probability $P_{wf}$, from wait state to fail state can be simply calculated as:

$$P_{wf} = 1 - P_{sw} - P_{ws} \quad (19)$$

Solving the equilibrium equations of the wireless node state transition diagram, the limiting probability of state succeed, $\Pi_s$ can be given by:

$$\Pi_s = \frac{P_{ws}}{2-P_{sw}}$$

$$= \frac{p \cdot (1-p)^M \cdot ((1-p)^p B(a_{data}))^{2L_{rts}}}{2 - (1-p)^M} \quad (20)$$

The value of $\Pi_s$ is substituted into Equation (12). Then the obtained value of $\Pi_s$ is substituted back into Equation (15) so that $\theta_i$, the ratio between successful transmission time to the total network time, can be derived. As illustrated in Section VII, the value of $\theta_i$ will be used to evaluate the total network throughput. Also, $\theta_i$ will be used to get the percentage of the total time consumed in collisions, hence, the energy consumption can be evaluated.

VI. ENERGY COMPUTATION

In addition to transmitting the RTS/CTS packets with high transmit power and the data packets with reduced power, transmission energy is also consumed in retransmitting control frames in case of collisions. To compute the total energy consumed in the network we first investigate the power consumption in data and control message transmissions. Second, we derive the time spent in successful transmission and that consumed during collisions.

Due to the free space power loss, as mentioned in Equation (1), the transmission power for data messages, $P_{data}$, can be simply given as shown:

$$P_{data} = C \cdot a_{data}^\gamma \quad (21)$$

where $C$ is a constant that depends on the wireless network interface card and $\gamma$ is the path loss factor.

Similar to the data frames, the power consumed in transmitting the RTS control frames is also proportional the transmission distance ($a_{rts}$) raised to the power of $\gamma$. However, retransmissions occur due to collisions with the RTS frames sent by other nodes. Hence, the power consumption in RTS transmission, $P_{rts}$, is given by:

$$P_{rts} = \sum_{i=1}^{M} \binom{M}{i} \cdot i \cdot C \cdot a_{rts}^\gamma \cdot p^i \cdot (1-p)^{M-i} \quad (22)$$

where $p$ is the probability that a node transmits at this time slot as given by Equation (10). $P_{rts}$ is the summation of the power consumed in sending $i$ RTS frames multiplied by the probability that $i$ nodes transmit an RTS frame at the same time slot, where $i$ ranges from 1 to $M$ and $M$ is the total number of nodes included in the sender coverage area.

Furthermore, $P_{cts}$, the power consumed in transmitting the CTS frame, takes the same form as $P_{rts}$. However, the number of nodes contending for accessing the wireless channel are those nodes hidden from the sender as illustrated by Figure 8. The number of hidden terminals, $K$, can be given as $p \cdot B(a_{data})$. Hence, $P_{cts}$ takes the form:

$$P_{cts} = \sum_{i=1}^{K} \binom{K}{i} \cdot i \cdot C \cdot a_{cts}^\gamma \cdot p^i \cdot (1-p)^{K-i} \quad (23)$$

By definition, $\theta_i$ in Equation(15) is the percentage of time the node is in successful data transmission to the total consumed time (the summation of transmission time and contention time). Hence the total consumed time, $T_{total}$, can be given as:

$$T_{total} = \frac{L_{rts} + L_{cts} + L_{data} + L_{ack}}{\theta_i} = \frac{T_{transmit}}{\theta_i} \quad (24)$$

Solving the equilibrium equations of the wireless channel state transition diagram, discussed in Section V, we can derive the percentage of time the system is in RTS-col relative to the total time, $\theta_r$, as:

$$\theta_r = \frac{\theta_i}{P_{lt}} \cdot P_{tr} \quad (25)$$

where $P_{lt}$ and $P_{tr}$ are given by Equations (12) and (13) respectively. Similarly, the percentage of time the system is in CTS-col relative to the total time, $\theta_c$, is:

$$\theta_c = \frac{\theta_i}{P_{lt}} \cdot P_{wc} \quad (26)$$
Hence the total contention time during collisions and control frame retransmissions has an RTS component, $T_{\text{RTS}} = \theta \cdot T_{\text{total}}$, and a CTS component, $T_{\text{CTS}} = \theta \cdot T_{\text{total}}$.

Having derived both the time and power consumption in transmitting the data frames and in the collision/retransmissions, we can simply evaluate the total energy consumption in the network, $E$, by multiplying the energy per hop by the expected number of hops, $\bar{L}/a_{\text{data}}$, in the network:

$$E = \frac{\bar{L}}{a_{\text{data}}} \cdot \left( P_{\text{data}} \cdot T_{\text{transmit}} + P_{\text{RTS}} \cdot T_{\text{RTS}} + P_{\text{CTS}} \cdot T_{\text{CTS}} \right)$$

(27)

As discussed in Section VII, using Equation (27) we can evaluate the total energy consumption in the network and also investigate the energy consumption per message for different node transmission ranges, and, thus, we determine the optimum transmission power for both the control and data messages based on the given network parameters.

VII. NUMERICAL RESULTS

Using the analytical equations previously derived and substituting the different network parameters by the values shown in Table I, we present results for the network throughput and the total energy consumption for a uniformly distributed adhoc network.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>RTS packet time</td>
<td>$L_{\text{RTS}}$</td>
<td>13 slot time</td>
</tr>
<tr>
<td>CTS packet time</td>
<td>$L_{\text{CTS}}$</td>
<td>12 slot time</td>
</tr>
<tr>
<td>Data packet time</td>
<td>$L_{\text{data}}$</td>
<td>287 slot time</td>
</tr>
<tr>
<td>Ack packet time</td>
<td>$t_{\text{ack}}$</td>
<td>12 slot time</td>
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<tr>
<td>Processing gain</td>
<td>$G$</td>
<td>10 db</td>
</tr>
<tr>
<td>SIR Threshold</td>
<td>$\text{SIR}_{\text{min}}$</td>
<td>21 db</td>
</tr>
<tr>
<td>Path loss factor</td>
<td>$\gamma$</td>
<td>2</td>
</tr>
<tr>
<td>Expected path length</td>
<td>$\bar{L}$</td>
<td>16 $d$</td>
</tr>
<tr>
<td>Contention window</td>
<td>CW</td>
<td>[16, 1024] slot time</td>
</tr>
<tr>
<td>Node density</td>
<td>$\rho$</td>
<td>[1, 3] node/$d^2$</td>
</tr>
</tbody>
</table>

The first five parameters are derived from the IEEE 802.11 specifications [14]. $\text{SIR}_{\text{min}}$ is set according to [28] for 10% Packet Error Rate (PER). $\gamma$ is set to 2 for the free space line of sight case and $\bar{L}$ is set to 16 (changing $\bar{L}$ will only have a linear effect on the results).

$\rho$ and CW are simulation parameters that are changed to investigate their effect on the network throughput and energy consumption; CW ranges from $CW_{\text{min}} = 16$ to $CW_{\text{max}} = 1024$ slot time [1]. Moreover, the unit of distance is taken to be an arbitrary unit of length $d$ in which the expected path length, the data transmission range ($a_{\text{data}}$), and the control frame transmission range ($a_{\text{RTS}}$) are given.

If we assume that the network is partitioned into several flows, where a flow is each node that can transmit at the same time without causing a collision, then the total network throughput can be defined as the sum of throughputs of each flow. We define $\sigma$ to denote the number of nodes that can concurrently transmit at the same time without causing a collision divided by the total number of network nodes. As discussed in Section IV, $\sigma$ can be defined as the total number of nodes in each interfering ring divided by the total number of network nodes. Hence, for a large network of radius $R$, $\sigma$ can be given as:

$$\sigma = \frac{1}{\rho \cdot \pi \cdot R^2} \cdot \sum_{i=1}^{\text{\#RTS}} 3 \cdot i$$

$$\approx \frac{3}{2 \cdot \rho \cdot \pi \cdot a_{\text{RTS}}^2}$$

(28)

where $\rho$ is the node density and the number of interference rings in the network is given by $R/a_{\text{RTS}}$.

Let $\mu$ be the traffic produced by each node in the network, expressed in messages/second. Thus, the total throughput per node can be simply obtained as the product of the average number of concurrently transmitting nodes, the “own” produced traffic per node, and the percentage of time the node is actually in a successful transmission status.

$$\text{Total Throughput per node} = \sigma \times \mu \times \theta_i$$

(29)

Figure 9 shows the results for the network throughput per node. These results emphasize the fact that for a given $a_{\text{RTS}}$ there is an optimal distance ($a_{\text{data}}$), by which the data packets should be sent in order to maximize the network throughput. It should be noted that, $a_{\text{data}} \leq a_{\text{RTS}}$ because the control frames are sent with a high power to prevent the “Hidden Terminal Jamming Problem”, as
previously mentioned. The lower bound on $a_{data}$ is a function of $\rho$ and determined such that there is at least one receiver in the transmission range of the sender.

As shown in Figure 9, at small $a_{data}$ the node is sending to a near neighbor, which increases the number of hops needed per message reducing the network throughput. As $a_{data}$ increases, the number of hops per message decreases and the throughput increases. For a given $a_{RTS}$ the maximum throughput is up to 30% higher than the throughput at the minimum value for $a_{data}$, this proves that it is not always optimal to use the minimum value for $a_{data}$ as proposed in previous work [7] [23]. However, at large $a_{data}$ the number of hidden terminals increases, leading to an increase in the number of collisions and a decrease in the network throughput.

On the other hand, the total network throughput degrades as $a_{RTS}$ increases. Increasing the $a_{RTS}$ reduces the interference level since more nodes defer their transmission when the data frame is being transmitted. But this effect seems to be overwhelmed by the collision effect as the number of colliding nodes trying to access the medium increases, causing an increased number of collisions of control messages and thus reduced throughput. This surprising result is contrary to the scheme proposed where $a_{RTS}$ is maximum and $a_{data}$ is minimum [7].

Figure 10 shows the results for the total network energy consumption. As $a_{data}$ increases, the energy consumed in data messages transmission dominates the total energy consumption. At large $a_{data}$ the number of hidden terminals from the sender increases and the energy wasted during CTS collision dominates the network energy consumption. Additionally, the message reaches its destination with fewer hops, but the energy per hop is high due to the $r^4$ factor in Equation (21).

By evaluating the energy consumption per message (that is, the energy normalized by the throughput) in the network, an interesting result is obtained. As shown in Figure 11 the energy consumption per message increases with larger $a_{RTS}$. However, the effect of $a_{data}$ is much less pronounced, leading to the choice of a slightly larger $a_{data}$ than the minimum, at the benefit of increasing throughput.

The results from Figures 9–11 show that the power by which the control frames are transmitted should be minimized to the level that just keep the network fully connected. Further, $a_{data}$ should not be necessarily set to the smallest possible value.

Figure 12 shows the effect of changing the node density on the network throughput. As expected, when the density (number of nodes) increases the throughput decreases since the number of collisions increase as more nodes are contending to access the wireless channel. However, the reduction in the throughput (e.g., the large drop between $\rho = 1$ and $\rho = 2$) is much larger than that reported by [9] since we take into account the combined effect of both the collision and interference.

The effect of changing the node density on the overall
energy consumption is shown in Figure 13. At a specific hop length (\(a_{data}\)) the number of nodes within the node coverage area increases with the increase of \(\rho\) and hence the number of contending nodes to access the wireless channel increases leading to an increase in the energy wasted during collision and retransmissions.

Figure 14 shows the effect of changing the contention window size on the network throughput. From Equation (10), with smaller CW the probability that a node transmits at the current slot time increases and hence the probability of collision increases. Thus, the smaller the CW, the lower the throughput; this suggests that the contention window should be set to a large initial value to increase throughput, despite the delays that this may incur. It should also be noted that as the CW decrease the optimal \(a_{data}\) approaches its minimum value, therefore, at smaller window size it is better to use the minimum data power between relay nodes.

The effect of changing the contention window size on the energy consumption is shown in Figure 15. When CW decreases, the probability that a node transmits at the current slot time increases and hence the probability of collision increases, causing more energy to be wasted during collision.

VIII. CONCLUSION

In our work we investigated the effect of transmission power control for power-aware adhoc networks on the overall throughput and energy savings in the network. We have shown that it is not always optimal to send the data packets to the nearest neighbor. However, for a given expected path length and a given node density, we derived expressions to compute the optimal transmission distance that will yield maximum throughput of the network and minimized energy consumption per message.

Furthermore, we proved that the control messages should not be sent with the maximum power as was suggested by previous work. By investigating the energy consumption per message, we were able to prove that the transmission power for control frames should be minimized to the extent of keeping the network connected.

Lastly, our work suggests that the contention window should be initialized to a larger value than currently suggested by protocol specifications.

We will extend our work in several ways. First, the idle energy consumption in the network and the energy consumed in the relay nodes during receiving the traffic should be investigated in addition to transmission energy. Second, the delays in the network should be accounted for when setting the transmission power for control and data frames. Third, studying the effect of changing the selection criteria of relay nodes on network lifetime is critical. The relay nodes may be selected based on different factors, such as their current battery capacity, in addition to their distance from the sender and the receiver.
REFERENCES